

# Static solutions of a 6-dimensional Einstein-Yang-Mills model

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## Abstract

We study the Einstein-Yang-Mills equations in a 6-dimensional space-time. We make a self-consistent static, spherically symmetric ansatz for the gauge fields and the metric. The metric of the manifold associated with the two extra dimensions contains off-diagonal terms. The classical equations are solved numerically and several branches of solutions are constructed. We also present an effective 4-dimensional action from which the equations can equally well be derived. This action is a standard Einstein-Yang-Mills-Higgs theory extended by three scalar fields. Two of the scalar fields are interpreted as dilatons, while the one associated with the off-diagonal term of the metric induces very specific interactions.

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## I. INTRODUCTION

In an attempt to unify electrodynamics and general relativity, Kaluza introduced an extra, a fifth dimension [1] and assumed all fields to be independent of the extra dimension. Klein [2] followed this idea, however, he assumed the fifth dimension to be compactified on a circle of Planck length. The resulting theory describes 4-dimensional Einstein gravity plus Maxwell's equations. One of the new fields appearing in this model is the dilaton, a scalar companion of the metric tensor. In an analogue way, this field arises in the low energy effective action of superstring theories and is associated with the classical scale invariance of these models [3].

Both string theories [4] as well as so-called “brane worlds” [5] assume that space-time possesses more than four dimensions. In string theories, these extra dimensions are -following the idea of Klein- compactified on a scale of the Planck length, while in brane worlds, which assume the Standard model fields to be confined on a 3-brane, they are large and even infinite. It should thus be interesting to study classical solutions of non-abelian gauge theories in higher dimensions.

An Einstein-Yang-Mills model in  $4 + 1$  dimensions was studied recently [6]. Assuming the metric and matter fields to be independent on the extra coordinates, an effective 4-dimensional Einstein-Yang-Mills-Higgs-dilaton model appears with one Higgs triplet and one scalar dilaton. This idea was taken further to  $4 + n$  dimensions [7], where  $n$  Higgs triplets and  $n$  dilatons appear. In contrast to one extra dimension, however, it appeared that in two or more extra dimensions, constraints result from the off-diagonal terms of the energy-momentum tensor. This leads to the fact that only solutions with either only one non-zero Higgs field or with all Higgs fields constant are allowed. In this paper, we study an Einstein-Yang-Mills model in  $4 + 2$  dimensions and we introduce an off-diagonal term in the metric associated with the extra dimensions in order to be able to obtain non-trivial solutions. We introduce the model in Section II and give the Ansatz and equations of motion in Section III. In Section IV, we give the 4-dimensional effective action from which the equations of motion can equally well be derived. In Section V, we present our numerical results and finally, in Section VI we give our conclusions.

## II. THE MODEL

The Einstein-Yang-Mills Lagrangian in  $d = 4 + 2$  dimensions is given by:

$$S = \int \left( \frac{1}{16\pi G_{(6)}} R - \frac{1}{4e^2} F_{MN}^a F^{aMN} \right) \sqrt{g^{(6)}} d^6x \quad (1)$$

with the SU(2) Yang-Mills field strengths  $F_{MN}^a = \partial_M A_N^a - \partial_N A_M^a + \epsilon_{abc} A_M^b A_N^c$ , the gauge index  $a = 1, 2, 3$  and the space-time index  $M = 0, \dots, 5$ .  $G_{(6)}$  and  $e$  denote respectively the 6-dimensional Newton's constant and the coupling constant of the gauge field theory.  $G_{(6)}$  is related to the Planck mass  $M_{pl}$  by  $G_{(6)} = M_{pl}^{-4}$  and  $e^2$  has the dimension of [length]<sup>2</sup>.

## III. ANSATZ AND EQUATIONS OF MOTION

In this paper, we will construct solutions with off-diagonal components of the metric tensor. The Ansatz for the metric then reads:

$$g_{MN} dx^M dx^N = e^{-(\xi_1 + \xi_2)} (-A^2 N dt^2 + N^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2) + \cosh\left(\frac{J}{2}\right) [e^{2\xi_1} (dx^4)^2 + e^{2\xi_2} (dx^5)^2] + 2e^{\xi_1 + \xi_2} \sinh\left(\frac{J}{2}\right) dx^4 dx^5, \quad (2)$$

where the functions  $A$ ,  $N$ ,  $J$ ,  $\xi_1$ ,  $\xi_2$  depend on the variable  $r$  only and  $N(r) = 1 - \frac{2m(r)}{r}$ .

The determinant of the metric is then given by:

$$\sqrt{-g^{(6)}} = A r^2 \sin \theta e^{-(\xi_1 + \xi_2)}. \quad (3)$$

The  $\xi_i$ ,  $i = 1, 2$  play the role of two scalar dilatons.

The Ansatz for the gauge field reads:

$$A_M^a dx^M = A_\mu^a dx^\mu + \sum_{k=1}^2 \Phi_k^a dy^k. \quad (4)$$

Note that the  $\Phi_j^a$ ,  $j = 1, 2$  play the role of Higgs fields.

The spherically symmetric Ansatz is given by [8]:

$$A_r^a = A_t^a = 0, \quad (5)$$

$$A_\theta^a = (1 - K(r)) e_\varphi^a, \quad A_\varphi^a = -(1 - K(r)) \sin \theta e_\theta^a, \quad (6)$$

$$\Phi_j^a = v H_j(r) e_r^a, \quad j = 1, 2, \quad (7)$$

where  $v$  is a mass scale determining the vacuum expectation values of the Higgs fields.

## A. Equations of motion

Using the metric (2), the matter Lagrangian  $L_{mat}$  reads:

$$L_{mat} = -\frac{1}{4}e^{2\xi_1+2\xi_2}F_{\mu\nu}^a F^{\mu\nu,a} - \frac{1}{2}\cosh\left(\frac{J}{2}\right)\left[e^{\xi_2-\xi_1}F_{\mu 4}^a F_4^{\mu,a} + e^{\xi_1-\xi_2}F_{\mu 5}^a F_5^{\mu,a}\right] \\ - \frac{1}{2}\sinh\left(\frac{J}{2}\right)F_{\mu 5}^a F_4^{\mu,a} - \frac{1}{2}e^{-2\xi_1-2\xi_2}F_{45}^a F_4^{5,a} \quad , \quad \mu, \nu = 0, 1, 2, 3 \quad . \quad (8)$$

The last term in (8) vanishes since

$$F_{45}^a F_{45}^a \propto (\Phi_1^a \times \Phi_2^a)^2 = 0$$

This is due to the fact that the fields  $\Phi_1, \Phi_2$  are assumed to be parallel.

With

$$F_{\mu(i+3)}^a = \partial_\mu \Phi_i^a + \varepsilon_{abc} A_\mu^b \Phi_i^c = D_\mu \Phi_i^a \quad , \quad i = 1, 2 \quad (9)$$

the matter Lagrangian  $L_{mat}$  reads:

$$L_{mat} = -\frac{1}{4}e^{2\xi_1+2\xi_2}F_{\mu\nu}^a F^{\mu\nu,a} \\ - \frac{1}{2}\cosh\left(\frac{J}{2}\right)e^{\xi_2-\xi_1}(D_\mu \Phi_1^a D^\mu \Phi_1^a + D_\mu \Phi_2^a D^\mu \Phi_2^a) \\ - \frac{1}{2}\sinh\left(\frac{J}{2}\right)D_\mu \Phi_1^a D^\mu \Phi_2^a \quad (10)$$

Inserting the Ansätze (6)-(7) and using

$$x = evr \quad , \quad \mu = evm \quad (11)$$

this reads:

$$L_{mat} = -e^{\xi_1+\xi_2}\left[B + \frac{D}{2} + \left(\frac{A_1}{2} + C_1 + \frac{A_2}{2} + C_2\right)\right] + A_{12} + 2C_{12} \quad (12)$$

with the abbreviations:

$$A_i = e^{-2\xi_i} N (H'_i)^2 \cosh\left(\frac{J}{2}\right) \quad , \quad i = 1, 2 \quad , \quad (13)$$

$$B = e^{\xi_1+\xi_2} \frac{N(K')^2}{x^2} \quad , \quad (14)$$

$$C_i = e^{-2\xi_i} \frac{K^2 H_i^2}{x^2} \cosh\left(\frac{J}{2}\right) \quad , \quad i = 1, 2 \quad , \quad (15)$$

$$D = e^{\xi_1+\xi_2} \frac{(K^2 - 1)^2}{x^4} \quad , \quad (16)$$

$$A_{12} = e^{-\xi_1-\xi_2} N H'_1 H'_2 \sinh\left(\frac{J}{2}\right) \quad , \quad (17)$$

$$C_{12} = e^{-\xi_1-\xi_2} \frac{K^2 H_1 H_2}{x^2} \sinh\left(\frac{J}{2}\right) \quad . \quad (18)$$

The non-vanishing components of the energy-momentum tensor are given by:

$$T_0^0 = -\frac{1}{2}e^{\xi_1+\xi_2} [2B + D + A_1 + 2C_1 + A_2 + 2C_2 - A_{12} - 2C_{12}] \quad (19)$$

$$T_1^1 = e^{\xi_1+\xi_2} \left[ B - \frac{D}{2} + \frac{A_1}{2} + \frac{A_2}{2} - C_1 - C_2 + A_{12} - 2C_{12} \right] \quad (20)$$

$$T_2^2 = T_3^3 = -e^{\xi_1+\xi_2} \left[ \frac{A_1}{2} + \frac{A_2}{2} - \frac{D}{2} - A_{12} \right] \quad (21)$$

$$T_4^4 = -e^{\xi_1+\xi_2} \left[ B + \frac{D}{2} - \frac{A_1}{2} + \frac{A_2}{2} - C_1 + C_2 \right] \quad (22)$$

$$T_5^5 = -e^{\xi_1+\xi_2} \left[ B + \frac{D}{2} + \frac{A_1}{2} - \frac{A_2}{2} + C_1 - C_2 \right] \quad (23)$$

$$T_4^5 = -e^{2\xi_1} \left[ \tanh\left(\frac{J}{2}\right) (A_1 + 2C_1) - \coth\left(\frac{J}{2}\right) (A_{12} + 2C_{12}) \right] \quad (24)$$

$$T_5^4 = -e^{2\xi_2} \left[ \tanh\left(\frac{J}{2}\right) (A_2 + 2C_2) - \coth\left(\frac{J}{2}\right) (A_{12} + 2C_{12}) \right] \quad (25)$$

The presence of a off-diagonal term in the metric renders the components of the Einstein tensor very lengthy and we do not give them explicitly here. We note that these off-diagonal terms have to be included for codimension larger than one to avoid solutions with trivial Higgs fields. This is a new feature with respect to [6].

Five independent Einstein equations can be obtained for the five metric functions parametrizing the metric (with  $\alpha^2 = 4\pi G v^2$ ):

$$\begin{aligned} \mu' &= \alpha^2 x^2 \left[ B + \frac{D}{2} + \frac{A_1}{2} + C_1 + \frac{A_2}{2} + C_2 - A_{12} - 2C_{12} \right] \\ &+ \frac{1}{32} x^2 N \left[ (J')^2 + 10(\xi'_1)^2 + 10(\xi'_2)^2 + 12\xi'_1\xi'_2 \right] \\ &+ \frac{1}{16} \cosh\left(\frac{J}{2}\right) x^2 N (\xi'_1 - \xi'_2)^2 \quad , \end{aligned} \quad (26)$$

$$\begin{aligned} A' &= \frac{\alpha^2 A x}{N} [2B + A_1 + A_2 - 2A_{12}] + \frac{1}{16} A x \left[ (J')^2 + 10(\xi'_1)^2 \right. \\ &\left. + 10(\xi'_2)^2 + 12\xi'_1\xi'_2 + 2 \cosh\left(\frac{J}{2}\right) x^2 N (\xi'_1 - \xi'_2)^2 \right] \quad , \end{aligned} \quad (27)$$

$$\begin{aligned} \xi_1'' &= -\tanh\left(\frac{J}{2}\right) J'(\xi'_1 - \xi'_2) - \xi'_1 \frac{1+N}{N} \\ &+ \alpha^2 \left[ \hat{T}_{22} \frac{2\xi'_1}{xN} + \hat{T}_{44} e^{-\xi_2-3\xi_1} \frac{\cosh(\frac{J}{2})(x\xi'_1 - 1) + x\xi'_1 - 3}{2N \cosh(\frac{J}{2})} \right. \\ &\left. + \hat{T}_{55} e^{-\xi_1-3\xi_2} \frac{\cosh(\frac{J}{2})(x\xi'_1 - 1) + x\xi'_1 + 1}{2N \cosh(\frac{J}{2})} + \hat{T}_{54} e^{-2\xi_1-2\xi_2} \frac{2 \sinh(\frac{J}{2})(1 - x\xi'_1)}{N} \right] \quad , \end{aligned} \quad (28)$$

$$\begin{aligned}
J'' = & (\xi_1' - \xi_2')^2 - J' \frac{1+N}{xN} \\
& + \alpha^2 \left[ \hat{T}_{22} \frac{2J'}{xN} + \hat{T}_{44} e^{-3\xi_1 - \xi_2} \frac{\cosh(\frac{J}{2}) x J' + 4 \sinh(\frac{J}{2})}{N} \right. \\
& \left. + \hat{T}_{55} e^{-3\xi_2 - \xi_1} \frac{\cosh(\frac{J}{2}) x J' + 4 \sinh(\frac{J}{2})}{N} - 2\hat{T}_{54} e^{-2\xi_2 - 2\xi_1} \frac{4 \cosh(\frac{J}{2}) + x J' \sinh(\frac{J}{2})}{N} \right], \quad (29)
\end{aligned}$$

where we use the abbreviation

$$\hat{T}_{MN} \equiv T_{MN} - \frac{1}{4} g_{MN} T_K^K, \quad M, N, K = 0, 1, 2, 3, 4, 5, \quad (30)$$

and the prime denotes the derivative with respect to  $x$ . Note that the equation for  $\xi_2$  can be obtained for (28) by exchanging  $\xi_1 \leftrightarrow \xi_2$ .

Finally the variation of the action (1) with respect to the matter fields leads to three differential equations for the functions  $K$ ,  $H_1$  and  $H_2$ :

$$\begin{aligned}
(K' e^{\xi_1 + \xi_2} AN)' = & AK \left[ e^{\xi_1 + \xi_2} \frac{(K^2 - 1)}{x^2} + \cosh\left(\frac{J}{2}\right) (e^{-2\xi_1} (H_1)^2 + e^{-2\xi_2} (H_2)^2) \right] \\
& - 2AH_1 H_2 K e^{-\xi_1 - \xi_2} \sinh\left(\frac{J}{2}\right), \quad (31)
\end{aligned}$$

$$\begin{aligned}
\frac{e^{2\xi_a}}{x^2 AN} (e^{-2\xi_a} AN x^2 H_a')' = & 2 \frac{1}{x^2 N} K^2 H_a + \frac{1}{2} e^{\xi_a - \xi_b} H_b' \left( J' - (\xi_a - \xi_b) \sinh\left(\frac{J}{2}\right) \right) \\
& - \frac{1}{2} \left( 1 - \cosh\left(\frac{J}{2}\right) \right) (\xi_a' - \xi_b') H_a', \quad (32)
\end{aligned}$$

where  $a, b = 1, 2$  and  $a \neq b$ .

## B. Boundary conditions

We will study globally regular, asymptotically flat solutions of the system above. This implies the following boundary conditions:

$$K(0) = 1, \quad H_j(0) = 0, \quad \mu(0) = 0, \quad \partial_x J|_{x=0} = 0, \quad \partial_x \xi_j|_{x=0} = 0, \quad j = 1, 2 \quad (33)$$

at the origin and

$$K(\infty) = 0, \quad H_j(\infty) = c_j, \quad A(\infty) = 1, \quad J(\infty) = 0, \quad \xi_j(\infty) = 0, \quad j = 1, 2 \quad (34)$$

at infinity.

#### IV. THE 4-DIMENSIONAL EFFECTIVE ACTION

The above equations can be obtained equally from the following 4-dimensional effective action:

$$\begin{aligned}
L_{mat} = & -\frac{1}{4}e^{2\kappa(\Psi_1+\Psi_2)}F_{\mu\nu}^a F^{\mu\nu,a} - \frac{1}{2}e^{-4\kappa\Psi_1} \cosh\left(\frac{J}{2}\right)(D_\mu\Phi_1^a)(D^\mu\Phi_1^a) \\
& - \frac{1}{2}e^{-4\kappa\Psi_2} \cosh\left(\frac{J}{2}\right)(D_\mu\Phi_2^a)(D^\mu\Phi_2^a) + e^{-2\kappa(\Psi_1+\Psi_2)} \sinh\left(\frac{J}{2}\right)(D_\mu\Phi_1^a)(D^\mu\Phi_2^a) \\
& - \frac{5}{12} [ (\partial_\mu\Psi_1)(\partial^\mu\Psi_1) + (\partial_\mu\Psi_2)(\partial^\mu\Psi_2) ] \\
& - \frac{1}{2}(\partial_\mu\Psi_1)(\partial^\mu\Psi_2) - \frac{1}{12} \cosh(J) [ (\partial_\mu\Psi_1)(\partial^\mu\Psi_1) + (\partial_\mu\Psi_2)(\partial^\mu\Psi_2) \\
& - 2(\partial_\mu\Psi_1)(\partial^\mu\Psi_2) ] - \frac{1}{32}(\partial_\mu J)(\partial^\mu J)
\end{aligned} \tag{35}$$

provided the following identification is done

$$\xi_i = 2\kappa\Psi_i \quad , \quad i = 1, 2 \quad , \quad \alpha^2 = 3\kappa^2 \quad . \tag{36}$$

Note that  $\kappa$  is a new coupling constant (the “dilaton” coupling) appearing in the effective action. The function  $J$  appears as a new dynamical scalar field in the effective action.

#### V. DISCUSSION OF NUMERICAL RESULTS

The system of non-linear differential equations can only be solved numerically. We have solved this system for numerous values of  $\alpha$ . For the numerical analysis we assume  $H_1(\infty) = H_2(\infty) = 1$ , but we believe that the pattern of solutions remains qualitatively equal for  $H_1(\infty) > H_2(\infty) > 0$ . In the limit  $\alpha = 0$  the metric is flat (Minkowski metric) (implying  $J(x) = \xi_1(x) = \xi_2(x) = 0$ ) and the solution of the equations is the 't Hooft-Polyakov magnetic monopole [8]. Due to the presence of two -linearly superposed- Higgs fields, the flat solution has mass  $\sqrt{2}$ , which is confirmed by our numerical results. For  $\alpha > 0$  the solution is progressively deformed by gravity, the fields  $J(x)$ ,  $\xi_1(x) = \xi_2(x) \equiv \xi(x)$ ,  $A(x)$ ,  $N(x)$  become non-trivial. In particular,  $N(x)$  develops a local minimum at some finite radius  $N(x_m) = N_m$ ,  $0 < A(0) < 1$  represents the minimum of the function  $A(x)$ , the dilaton field  $\xi(x)$  is negative and monotonically increasing, while the function  $J(x)$  is positive and decreases monotonically from  $J(0) > 0$  to  $J(\infty) = 0$ . The profiles of the different functions

is presented in Fig.1 for two different values of  $\alpha$ . This figure shows that  $|J(x)|$  is larger than  $|\xi_1| = |\xi_2| \equiv |\xi(x)|$ .

The values  $\xi(0), J(0)$  and  $A(0), N_m$  are plotted as functions of  $\alpha$  in Fig. 2 and Fig. 3, respectively. The branch which arises from the gravitational deformation of the flat monopole is indexed by the label “1” in the figures. In Fig.2 we also plot the ADM mass of the solution, which decreases with increasing  $\alpha$ .

The figures further demonstrate that the branch “1” of solutions does not exist for arbitrary large values of  $\alpha$ . Rather, the solutions exist only for  $0 \leq \alpha \leq \alpha_{max}$  with  $\alpha_{max} \approx 0.859$ . In addition, another branch of solutions exists for  $\alpha \leq \alpha_{max}$ . More precisely, we have managed to construct a second branch of solution (labelled “2” in the figures) for  $\alpha_1 \leq \alpha \leq \alpha_{max}$  (with  $\alpha_1 \approx 0.235$ ). Again, starting from  $\alpha_1$  and increasing  $\alpha$ , our numerical analysis strongly suggests the existence of a third branch labelled “3”. Note that the ADM mass of the solutions of branch “3” is very close to that of the solutions of branch “2”. This is why the two branches cannot be distinguished on the plot as far as their ADM mass is concerned. Continuing this construction, there probably exist a large (possibly infinite) number of branches on smaller and smaller intervals concentrated around the value  $\alpha \approx 0.288$ . This pattern is very reminiscent to the one occurring in the case with  $d = 4 + 1$  first discussed by Volkov [6]. However the behaviour of the mass of the solutions in the  $d = 4 + 1$  case and in the  $d = 4 + 2$  cases is considerably different. Indeed, for  $d = 4 + 1$ , the mass of the solutions on the branches “2”, “3” is higher than the mass of the solution on the branch “1”; in the present case, it is the contrary. Said in other words, for  $\alpha_1 \leq \alpha \leq \alpha_{max}$  the solution with the lowest mass is the one of branch “2”.

At first glance this result may appear paradoxal, since it is known that the monopole is topologically stable. However, we believe that this peculiar behaviour of the mass is deeply connected to the direct coupling between the two Higgs fields in the effective action. Roughly speaking the two Higgs fields are coupled with a factor  $-\sinh(J)$  (remember that this term is absent in absence of gravity  $J(x) = 0$ ). For  $J > 0$  it turns out that the three contributions due to the terms quadratic in the derivative of the Higgs fields have a tendency to compensate. For the solutions of the second branch, the dilatons  $\xi_1(x) = \xi_2(x)$  and the function  $J(x)$  deviate stronger from their flat space-time values than on the branch “1”. Specifically,  $J(x)$  becomes large enough to diminish considerably the contribution of the Higgs field to the mass. As a consequence, the mass becomes smaller than the mass on the



first branch.

Analyzing this result from the point of view of catastrophe theory it is tempting to conclude that the solutions on the branch “1” are unstable (they are sphaleron-like) and it is very likely that the monopole loses its non-trivial topology and its status of being a local minimum when it is coupled to the other fields appearing in the 4-dimensional effective model. There are  $k$  negative -unstable- modes while the solution of branch “2” have  $k - 1$  unstable modes. A direct analysis of the stability should definitely confirm this conclusion and provide the value of  $k$ .

## VI. CONCLUSIONS

The dimensional reduction of the Einstein-Yang-Mills theory from  $4 + n$  dimensions down to the standard 4 dimensions is an interesting problem. The main source of uncertainty is the way the fields depend on the extra dimensions. One can adopt, for the simplest case, the point of view that, in an appropriate gauge and with appropriate variables, the gauge and metric fields are independent on the extra coordinates. A 4-dimensional effective action which encodes all the effects of the extra dimension into a more or less “conventional” field theory can then be constructed.

In the case of  $n = 1$ , it was shown that the effective action is a Georgi-Glashow model appropriately coupled to one dilaton field [6]. Here we have studied in details the case  $n = 2$ . One of the main differences in comparison to the  $n = 1$  case is that the Einstein-Yang-Mills equations are consistent only if the metric has off-diagonal terms in the subspace of the extra dimensions. The corresponding effective action is a standard Einstein-Yang-Mills action supplemented by two Higgs triplets and three scalar fields, originating from the parametrization of the metric in the codimensions. The function corresponding to the off-diagonal term of the metric plays a very important role in the effective action, largely determining an interaction between the two Higgs fields. All these fields acquire kinetic parts as well as specific interactions.

The equations of motion can then be studied numerically. We have constructed several branches of classical solutions which can be seen as deformations of a magnetic monopole.

Several possibilities of extension of the result can be investigated, namely (1) the construction of non-abelian black hole, (2) the stability analysis of the different branches of regular

solutions, (3) the addition of higher order terms in the Yang-Mills action (“Born-Infeld” terms) and (4) the construction of axially symmetric solutions.

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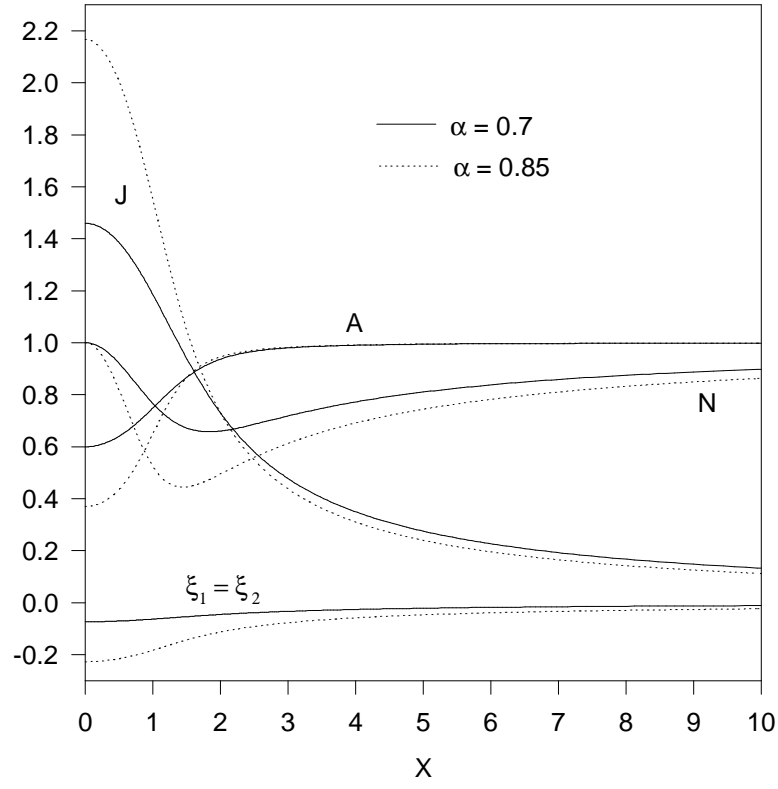


FIG. 1: The profiles of the metric functions  $A(x)$ ,  $J(x)$ ,  $N(x)$ ,  $\xi_1(x) = \xi_2(x) \equiv \xi(x)$  are shown for two different values of  $\alpha$ .

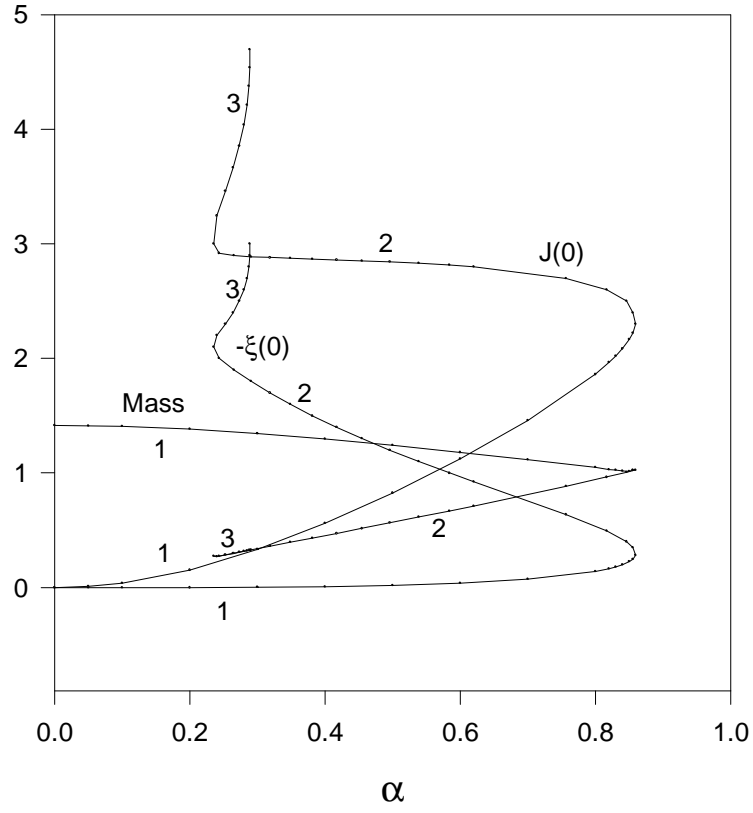


FIG. 2: The dependence of the values  $J(0)$  and  $-\xi_1(0) = -\xi_2(0) \equiv -\xi(x)$  as well as the ADM mass are shown as functions of  $\alpha$ .

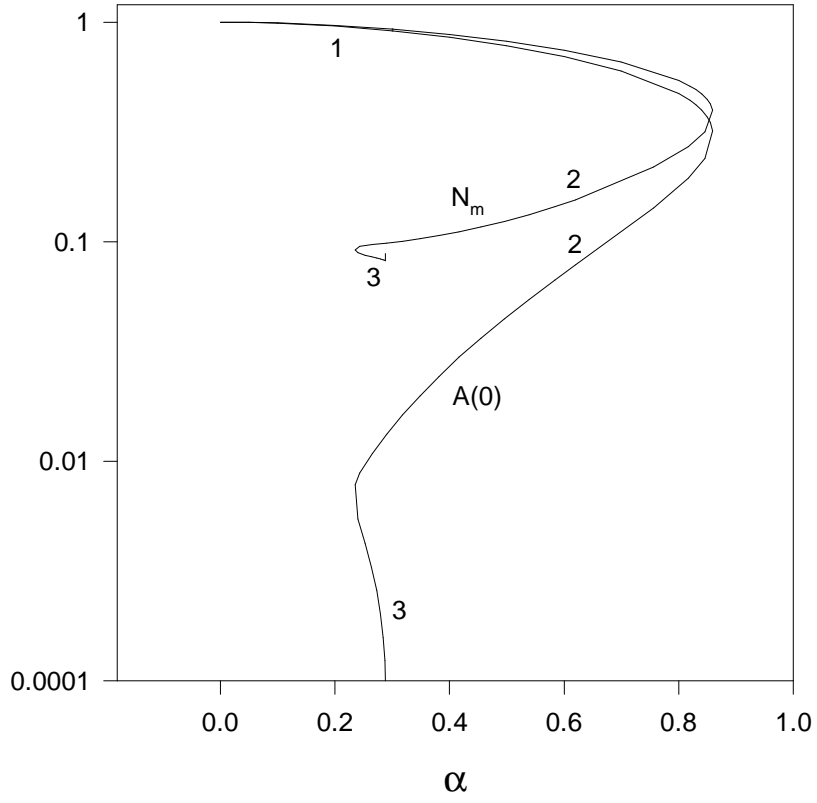


FIG. 3: The dependence of  $N(x_{min}) = N_m$  and  $A(0)$  are shown as functions of  $\alpha$ .